

# Some Common Probability Distributions

Distribution of $X$		Probability mass/density function	$E(X)$	$\text{Var}(X)$	Moment generating function $M_X(t)$
Bernoulli where $0 < p < 1$	$\text{Ber}(p)$	$p^x(1-p)^{1-x}$ for $x = 0, 1$	$p$	$p(1-p)$	$pe^t + 1 - p$ for $t \in \mathbb{R}$
Binomial where $n \in \mathbb{N}, 0 < p < 1$	$\text{B}(n, p)$	$\binom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, 2, \dots, n$	$np$	$np(1-p)$	$(pe^t + 1 - p)^n$ for $t \in \mathbb{R}$
Geometric where $0 < p < 1$	$\text{Geo}(p)$	$(1-p)^{x-1}p$ for $x = 1, 2, 3, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$ for $t < -\ln(1-p)$
Negative binomial where $r \in \mathbb{N}, 0 < p < 1$	$\text{NB}(r, p)$	$\binom{x-1}{r-1} (1-p)^{x-r} p^r$ for $x = r, r+1, r+2, \dots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[ \frac{pe^t}{1-(1-p)e^t} \right]^r$ for $t < -\ln(1-p)$
Hypergeometric where $N \in \mathbb{N}, m, n \in \{0, \dots, N\}$	$\text{Hyp}(N, m, n)$	$\frac{\binom{m}{x} \binom{N-m}{n-x}}{\binom{N}{n}}$ for $x \in \{\max(0, n+m-N), \dots, \min(n, m)\}$	$\frac{nm}{N}$	$\left(\frac{N-n}{N-1}\right) n \left(\frac{m}{N}\right) \left(1 - \frac{m}{N}\right)$	
Poisson where $\lambda > 0$	$\text{Po}(\lambda)$	$\frac{e^{-\lambda} \lambda^x}{x!}$ for $x = 0, 1, 2, \dots$	$\lambda$	$\lambda$	$e^{\lambda(e^t-1)}$ for $t \in \mathbb{R}$
Uniform where $a, b \in \mathbb{R}, a < b$	$\text{U}(a, b)$	$\frac{1}{b-a}$ for $a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
Exponential where $\lambda > 0$	$\text{Exp}(\lambda)$	$\lambda e^{-\lambda x}$ for $x > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}$ for $t < \lambda$
Gamma where $\alpha, \lambda > 0$	$\Gamma(\alpha, \lambda)$	$\frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$ for $x > 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^\alpha$ for $t < \lambda$
Normal where $\mu \in \mathbb{R}, \sigma > 0$	$\text{N}(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ for $x \in \mathbb{R}$	$\mu$	$\sigma^2$	$e^{\mu t + \frac{1}{2}\sigma^2 t^2}$ for $t \in \mathbb{R}$
Chi-squared where $r \in \mathbb{N}$	$\chi^2(r)$	$\frac{1}{\Gamma\left(\frac{r}{2}\right) 2^{\frac{r}{2}}} x^{\frac{r}{2}-1} e^{-\frac{x}{2}}$ for $x > 0$	$r$	$2r$	$\frac{1}{(1-2t)^{\frac{r}{2}}}$ for $t < \frac{1}{2}$
Student's $t$ where $r \in \mathbb{N}$	$t(r)$	$\frac{\Gamma\left(\frac{r+1}{2}\right)}{\sqrt{r\pi\Gamma\left(\frac{r}{2}\right)}} \left(1 + \frac{x^2}{r}\right)^{-\frac{r+1}{2}}$ for $x \in \mathbb{R}$	0 for $r > 1$	$\frac{r}{r-2}$ for $r > 2$	
Snedecor's $F$ where $m, n \in \mathbb{N}$	$F(m, n)$	$\frac{\Gamma\left(\frac{m+n}{2}\right)}{\Gamma\left(\frac{m}{2}\right) \Gamma\left(\frac{n}{2}\right)} \left(\frac{m}{n}\right)^{\frac{m}{2}} x^{\frac{m}{2}-1} \left(1 + \frac{m}{n}x\right)^{-\frac{m+n}{2}}$ for $x > 0$	$\frac{n}{n-2}$ for $n > 2$	$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$ for $n > 4$	
Beta where $\alpha, \beta > 0$	$\text{Beta}(\alpha, \beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$ for $0 < x < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	

Notes:  $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$  for  $\alpha > 0$ ;  $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$ ,  $\Gamma(1) = 1$ ,  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ .